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$\Delta S = 1, 2$ Effective Weak Chiral Lagrangian from the Instanton Vacuum

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Abstract

We study the effective weak chiral Lagrangian from the instanton vacuum. We incorporate the effective weak Hamiltonian into the effective low-energy partition function defining the chiral symmetric quark-Goldstone boson interactions with the momentum-dependent dynamical quark mass. Employing the derivative expansion and the $1/N_c$ expansion, we derive the corresponding bosonic weak effective Lagrangian in leading order with the low energy constants to be used *e.g.* in chiral perturbation theory. We find that the momentum-dependent dynamical quark mass plays an essential role in improving the low energy constants and their ratio $g_{\underline{8}}/g_{\underline{27}}$.

Keywords: Instanton vacuum, Effective chiral Lagrangians, Kaon nonleptonic decays, $\Delta T = 1/2$ rule, Derivative expansion.

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I. INTRODUCTION

Understanding of the nonleptonic decays of light hadrons in the Standard Model (SM) has been one of the most difficult issues. While the SM explains the weak processes involving the change of strangeness by considering W exchange, the description of nonleptonic decays of light hadrons is rather complicated on account of the strong interactions in the low-energy regime (below 1 GeV). The problem is typified by the $\Delta T = 1/2$ selection rule, best known as the fact that the isosinglet amplitude of the $K \to \pi\pi$ decay dominates over the $T = 2$ amplitude by about 22 times. Despite a great deal of effort this dominance of the $\Delta T = 1/2$ channel over the $\Delta T = 3/2$ it has not been explained in a satisfactory way. The effective weak Hamiltonian derived from evolving the simple W-exchange-vertex from a scale of 80 GeV down to 1 GeV [1, 2, 3, 4, 5, 6, 7, 8, 9] presents a part of the answer, showing that the perturbative gluons inherent in the Wilson coefficients enhance the $\Delta T = 1/2$ channel. However, the perturbative gluons alone are not enough to explain the $\Delta T = 1/2$ rule completely. Thus, we need to consider other sources of the $\Delta T = 1/2$ enhancement. Since the structure of the light hadrons intimates already the importance of nonperturbative QCD, it is natural to consider its significance in describing low energy processes such as $K \to \pi\pi$ decay.

Recently, we have investigated the effective $\Delta S = 1, 2$ weak chiral Lagrangian to order $\mathcal{O}(p^4)$ within the framework of the chiral quark model (χQM) [10], focusing on determining the low energy constants (LECs) in the effective weak chiral Lagrangian. However, the ratio of the g_8/g_{27} obtained from the χQM deviated from the phenomenological values obtained in chiral perturbation theory (χPT). There was a suggestion to include the gluon condensate which is of order $\mathcal{O}(\alpha_s N_c)$ [11]. However, Ref. [11] did not consider the $\mathcal{O}(\alpha_s N_c)^2$ corrections which might be of significance at the same extent to the $\mathcal{O}(\alpha_s N_c)$ corrections.

In the present work, we want to improve our former study [10], based on the more general effective low-energy QCD partition function derived from the instanton vacuum which pertains to nonperturbative QCD. The main feature lies in the fact that the coupling strength of the chiral symmetric quark-Goldstone interactions is momentum-dependent. In fact, switching off the momentum dependence of the coupling and adding an appropriate regularization scheme will lead to the usual χQM used in the former investigation [10]. Due to the complexity of the formalism in the present approach we first concentrate on order $\mathcal{O}(p^2)$ and a part of $\mathcal{O}(1/N_c)$.

As far as the large N_c expansion is concerned, while it explains the strong-interaction sector quantitatively well, the nonleptonic weak interactions defy any explanation from the strict large N_c limit. Because of the fact that the $\Delta T = 1/2$ enhancement is badly underestimated in lowest oder in the large N_c expansion, it is inevitable to go beyond the leading order (LO) in $1/N_c$ [22, 23, 24, 25, 26, 27]. Moreover, if one expects a large contribution from the next-to-leading order (NLO) in $1/N_c$, a convergence problem for the $1/N_c$ expansion may occur [26]. In this case, one has to consider higher order corrections in $1/N_c$. Furthermore, since there are many different sources of nonleading-order corrections in $1/N_c$, one has to carefully analyze $\mathcal{O}(1/N_c)$ corrections. However, dealing with higher order corrections and all contributions of $\mathcal{O}(1/N_c)$ is beyond our work. Thus, we will restrict ourselves in this work to a part of the $1/N_c$ corrections: The $1/N_c$ diagrams from the quark operators.

The instanton vacuum elucidates one of the most important low-energy properties of QCD, *i.e.* the mechanism of spontaneous breaking of chiral symmetry [12, 13, 14]. The

Banks-Casher relation tells that the spectral density $\nu(\lambda)$ of the Dirac operator at zero modes is proportional to the chiral condensate known as an order parameter of spontaneous breaking of chiral symmetry:

$$\langle\bar{\psi}\psi\rangle \;=\; -\frac{\pi\nu(0)}{V^{(4)}}. \tag{1}$$

The picture of the instanton vacuum provides a good realization of spontaneous breaking of chiral symmetry. A finite density of instantons and antiinstantons produces the nonvanishing value of $\nu(0)$, which triggers the mechanism of chiral symmetry breaking. The Euclidean quark propagator in the instanton vacuum acquires the following form with a momentum-dependent quark mass generated dynamically, identified with the coupling strength between quarks and Goldstone bosons:

$$S_F(k) \;=\; \frac{\not{k} + iM(k)}{k^2 + M^2(k)} \tag{2}$$

with

$$M(k) \;=\; \mathrm{const}\cdot\sqrt{\frac{N\pi^2\bar{\rho}^2}{VN_c}}F^2(k\bar{\rho}). \tag{3}$$

The ratio N/V denotes the instanton density at equilibrium and the $\bar{\rho}$ is the average size of the instanton. The form factor $F(k\bar{\rho})$ is related to the Fourier transform of the would-be zero fermion mode of individual instantons. As will be shown later, the momentum dependence of the constituent quark plays an essential role in improving our previous results [10] concerning the effective weak chiral Lagrangian. It is in line with the recent works on the pion wave functions [15, 16] and skewed parton distribution [17], where the momentum-dependent quark mass plays a crucial role as well.

The instanton vacuum induces effective $2N_f$-fermion interactions [14, 18]. For example, it has a type of the Nambu-Jona-Lasinio model for $N_f = 2$ while for $N_f = 3$ it exhibits the 't Hooft determinant [19]. Goldstone bosons appear as collective excitations by quark loops generating a dynamic quark mass. Eventually it is found that at low energies QCD is reduced to an interacting quark-Goldstone boson theory given by the following Euclidean partition function [20]

$$\mathcal{Z} = \int \mathcal{D}\psi\mathcal{D}\psi^\dagger\mathcal{D}\pi^a \exp\left[\int d^4x\psi_f^{\dagger\alpha}\left(i\not{\partial} + i\sqrt{M(-i\partial)}U^{\gamma_5}\sqrt{M(-i\partial)}\right)_{fg}\psi_g^\alpha\right], \tag{4}$$

where U^{γ_5} stands for the pseudo-Goldstone boson:

$$U^{\gamma_5}(x) = U(x)\frac{1+\gamma_5}{2} + U^\dagger(x)\frac{1-\gamma_5}{2} \;=\; \exp\left(i\pi^a(x)\lambda^a\gamma_5\right). \tag{5}$$

The α is the color index, $\alpha = 1,\cdots,N_c$ and f and g are flavor indices. $M(-i\partial)$ is the constituent quark mass being now momentum-dependent. It will play a main role in the present work. This effective theory of quarks and light Goldstone mesons applies to quark momenta up to the inverse size of the instanton, $\bar{\rho}^{-1} \simeq 600$ MeV, which may act as a scale of the model ($\mu_{\chi\mathrm{QM}}$). A merit to derive the χQM from the instanton vacuum lies in the fact that the scale of the model is naturally determined by $\bar{\rho}^{-1}$. Furthermore, mesons and baryons can be treated on the same footing in the χQM. For example, the model has been very successful in describing the properties of the baryons [21].

We want to mention that there is a problem related to the gauge invariance in Eq.(4). Due to the nonlocality of the interaction expressed in Eq.(4), the vector and axial-vector Ward-Takahashi identities are not satisfied, that is, the conservation of the vector current (CVC) and the partial conservation of the axial-vector current(PCAC) do not hold. Thus, one has to make the effective action gauge-invariant. A well-known prescription to solve this problem is to insert the path-ordered exponent into the effective action [28, 29, 30, 31, 32, 33]. At the scale of the W boson, the charged-current weak interaction of the quarks is mediated by the W boson only and thus is a product of two local currents. However, when the charged-current weak interaction is scaled down to the hadronic scale, *i.e.* $\mu = 1$ GeV, the renormalization of the QCD quantum corrections make the current-current operator mixed with other different types of twist-four local composite operators [9]. However, in the strict large N_c limit, the contribution of all other composite operators is suppressed by $1/N_c$ except for the current-current one which is the same as that at the W scale. Thus, it need not be renormalized and the conservation of the currents must be taken into account.

The outline of the present paper is as follows: In Section 2 we sketch the instanton-induced chiral quark model, emphasizing in particular the momentum dependence of the constituent quark mass and explain how to perform the derivative expansion in the presence of the momentum-dependent constituent quark mass. In section 3, we show the basic formalism to obtain the effective weak chiral Lagrangian starting from the effective weak quark Hamiltonian. In section 4, we dicuss the large N_c limit in the nonleptonic weak Hamiltonian and the conserved currents with the nonlocal interaction. Section 5 is devoted to the derivation of the effective weak chiral Lagrangian to order $\mathcal{O}(p^2)$ and $\mathcal{O}(1/N_c)$. In Section 6 we discuss results. The conclusions and outlook are given in Section 7.

II. EFFECTIVE CHIRAL THEORY FROM THE INSTANTON VACUUM

The effective low-energy QCD partition function in Euclidean space can be written as

$$\mathcal{Z} = \int \mathcal{D}\psi\mathcal{D}\psi^\dagger\mathcal{D}\pi^a \exp\int d^4x\Big[\psi_f^{\dagger\alpha}(x)i\partial\!\!\!/\psi_f^\alpha(x) + i\int\frac{d^4kd^4l}{(2\pi)^8}e^{i(k-l)\cdot x}\sqrt{M(k)M(l)}\psi_f^{\dagger\alpha}(k)\left(U^{\gamma_5}\right)_{fg}\psi_g^\alpha(l)\Big], \tag{6}$$

The $M(k)$ is the momentum-dependent constituent quark mass expressed as follows:

$$M(k) = M_0F^2(k/\Lambda). \tag{7}$$

If we choose the $F(k/\Lambda)$ to be constant and add a regularization (*e.g.* Pauli-Villars or proper-time), the partition function becomes just that of the usual χQM. The original expression for the $F(k/\Lambda)$ [12, 13], which is obtained from the Fourier transformation of the would-be zero fermion mode of individual instantons with the sharp instanton distribution assumed, is as follows:

$$F(k/\Lambda) = 2z\left(I_0(z)K_1(z) - I_1(z)K_0(z) - \frac{1}{z}I_1(z)K_1(z)\right). \tag{8}$$

Here I_0, I_1, K_0, and K_1 denote the modified Bessel functions, z is defined as $z = k/(2\Lambda)$ and the cutoff parameter Λ is in this case just the inverse of $\bar{\rho}$. When k goes to infinity, $F(k/\Lambda) = F(k\bar{\rho})$ becomes

$$F(k\bar{\rho}) \longrightarrow \frac{6}{(k\bar{\rho})^3}. \tag{9}$$

Actually, the momentum-dependent quark mass is related to the nonlocal regularization in Euclidean space. There are other ways of understanding the nonlocal effective interaction without relying on the instanton vacuum [34]. In those methods, the momentum-dependent quark mass as a regularization appears as delocalizing the quark fields. So, various types of the $M(k)$ as a regulator with the regularization parameter Λ has been used by different authors [15, 16, 35].

Therefore, we will not confine ourselves to the expression given in Eq.(8) but rather try three different types of the $M(k)$:

$$M(k) = \begin{cases} \text{Eqs.}(7,\ 8) \\ M_0 \frac{\Lambda^4}{(\Lambda^2+k^2)^2} \\ M_0 \exp\left(-\frac{k^2}{\Lambda^2}\right) \end{cases} . \tag{10}$$

The $M(k)$ is normalized to M_0 at $k = 0$. Originally, $M(0)$ is taken to be around 350 MeV corresponding to the values of the following parameters: $\bar{R} \simeq 1$ fm and $\bar{\rho} \simeq 0.35$ fm. The $\bar{R}$ is the average distance between neighboring instantons. However, we will regard M_0 as a free parameter ranging from 200 MeV to 450 MeV and fit for each M_0 the parameter Λ to the pion decay constant $f_\pi = 93$ MeV. Figure 1 shows the momentum dependence of the three different types of $M(k)$ with $M_0 = 350$ MeV.

Having integrated out the quark fields in the partition function, we obtain

$$\mathcal{Z} = \int \mathcal{D}\pi^a \exp\left(-S_{\rm eff}[\pi^a]\right), \tag{11}$$

where the $S_{\rm eff}[\pi^a]$ stands for the effective chiral action:

$$S_{\rm eff}[\pi^a] = -N_c \ln \det D(U^{\gamma_5}). \tag{12}$$

Here, the $D(U^{\gamma_5})$ is the Dirac operator defined by

$$D = i\partial\!\!\!/ + i\sqrt{M(-i\partial)}U^{\gamma_5}\sqrt{M(-i\partial)}. \tag{13}$$

The Dirac operator is not Hermitian, so that it is useful to divide the effective action into the real and imaginary parts:

$$\begin{aligned} {\rm Re}S_{\rm eff} &= \frac{1}{2}\left(S_{\rm eff} + S^*_{\rm eff}\right) = -\frac{1}{2}N_c \ln\det\left[D^\dagger D\right], \\ i{\rm Im}S_{\rm eff} &= \frac{1}{2}\left(S_{\rm eff} - S^*_{\rm eff}\right) = -\frac{1}{2}N_c \ln\det\left[D/D^\dagger\right]. \end{aligned} \tag{14}$$

In order to calculate the real part, we substract the vacuum part and use the derivative expansion. We therefore write

$$\begin{aligned} {\rm Re}S_{\rm eff}[\pi^a] - {\rm Re}S_{\rm eff}[0] &= -\frac{N_c}{2}{\rm Tr}\ln\left(\frac{D^\dagger D}{D_0^\dagger D_0}\right) \\ &= -\frac{N_c}{2}\int d^4x \int \frac{d^4k}{(2\pi)^4} e^{-ikx}{\rm tr}\ln\left(\frac{D^\dagger D}{D_0^\dagger D_0}\right)e^{ikx} \\ &= -\frac{N_c}{2}\int d^4x \int \frac{d^4k}{(2\pi)^4}{\rm tr}\ln\left(\frac{D^\dagger(\partial\to\partial+ik)D(\partial\to\partial+ik)}{D_0^\dagger(\partial\to\partial+ik)D_0(\partial\to\partial+ik)}\right). \end{aligned} \tag{15}$$

Here we used a complete set of plane waves for the calculation of the functional trace, summed over all states and took the trace in x. 'tr' then denotes the usual matrix trace over flavor and Dirac indices. The rhs. of Eq.(15) can now be expanded in powers of the derivatives of the pseudo-Goldstone boson fields, $\partial\!\!\!/ U^{\gamma_5}$, and of $2ik\cdot\partial+\partial^2$. After some manipulation using

$$
\begin{aligned}
&D^\dagger(\partial\to\partial+ik)D(\partial\to\partial+ik) = \left(i\partial\!\!\!/ - k\!\!\!/ - i\sqrt{M(-i\partial+k)}U^{-\gamma_5}\sqrt{M(-i\partial+k)}\right)\\
&\quad\times\left(i\partial\!\!\!/ - k\!\!\!/ + i\sqrt{M(-i\partial+k)}U^{\gamma_5}\sqrt{M(-i\partial+k)}\right)\\
=&\;\; -\partial^2+k^2-2ik\cdot\partial+\sqrt{M(-i\partial+k)}U^{-\gamma_5}M(-i\partial+k)U^{\gamma_5}\sqrt{M(-i\partial+k)}\\
-&\;\; \sqrt{M(-i\partial+k)}\,(\partial\!\!\!/ U^{\gamma_5})\,\sqrt{M(-i\partial+k)}\\
=&\;\; -\partial^2+k^2-2ik\cdot\partial+M^2-M\,(\partial\!\!\!/ U^{\gamma_5})-2iM\tilde{M}'k_\alpha\left[2\partial_\alpha+U^{-\gamma_5}\,(\partial_\alpha U^{\gamma_5})\right]\\
-&\;\; M\tilde{M}'\left[2\partial^2+U^{-\gamma_5}\left(\partial^2U^{\gamma_5}\right)+2U^{-\gamma_5}\,(\partial_\alpha U^{\gamma_5})\,\partial_\alpha\right]\\
-&\;\; 2M\tilde{M}''k_\alpha k_\beta\left[2\partial_\alpha\partial_\beta+U^{-\gamma_5}\,(\partial_\alpha\partial_\beta U^{\gamma_5})+2U^{-\gamma_5}\,(\partial_\alpha U^{\gamma_5})\,\partial_\beta\right]\\
-&\;\; 2\tilde{M}'^2k_\alpha k_\beta\left[\left(\partial_\alpha U^{-\gamma_5}\right)(\partial_\beta U^{\gamma_5})+U^{-\gamma_5}\,(\partial_\alpha\partial_\beta U^{\gamma_5})+2U^{-\gamma_5}\,(\partial_\alpha U^{\gamma_5})\,\partial_\beta+2\partial_\alpha\partial_\beta\right]\\
+&\;\; i\tilde{M}'k_\alpha\Big[\,(\partial_\alpha\partial\!\!\!/ U^{\gamma_5})+2\,(\partial\!\!\!/ U^{\gamma_5})\,\partial_\alpha\Big]+\mathcal{O}(\partial^3),
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
D_0^\dagger\;&(\partial\to\partial+ik)D_0(\partial\to\partial+ik) = -\partial^2+k^2+M^2-2ik\cdot\partial\\
&-4iM\tilde{M}'k\cdot\partial-4\tilde{M}'^2k_\alpha k_\beta\partial_\alpha\partial_\beta-2M\tilde{M}'\partial^2-4M\tilde{M}''k_\alpha k_\beta\partial_\alpha\partial_\beta+\mathcal{O}(\partial^3),
\end{aligned}
\tag{17}
$$

where

$$
M=M(k),\quad \tilde{M}'=\frac{1}{2k}\frac{dM(k)}{dk},\quad \tilde{M}''=\frac{1}{4k^3}\left(\frac{d^2M(k)}{dk^2}k-\frac{dM(k)}{dk}\right),
\tag{18}
$$

we obtain eventually the effective chiral action with the momentum-dependent quark mass:

$$
\mathrm{Re}S_{\mathrm{eff}}[\pi^a]-\mathrm{Re}S_{\mathrm{eff}}[0] \;=\; \frac{f_\pi^2}{4}\int d^4x\,\langle L_\mu L_\mu\rangle\,.
\tag{19}
$$

In Eq.(19) $\langle\rangle$ denotes the flavor trace, $L_\mu=iU^\dagger\partial_\mu U$ is a Hermitian $N_f\times N_f$ matrix and f_π denotes the pion decay constant:

$$
f_\pi^2 \;=\; 4N_c\int\frac{d^4k}{(2\pi)^4}\frac{M^2(k)-\frac{1}{2}M(k)M'(k)k+\frac{1}{4}M'^2(k)k^2}{(k^2+M^2(k))^2}
\tag{20}
$$

with

$$
M'(k) \;=\; \frac{dM(k)}{dk}.
\tag{21}
$$

When we switch off the momentum dependence of the constituent quark mass, we end up with the former expression for f_π^2 [10]:

$$
f_\pi^2 \;=\; 4N_c^2\int\frac{d^4k}{(2\pi)^4}\frac{M^2}{(k^2+M^2)^2},\; M=\;\mathrm{const.}
\tag{22}
$$

which is logarithmically divergent. The quark condensate, which is just the trace of the quark propagator given by Eq.(2), is written by

$$
\langle\bar\psi\psi\rangle_M \;=\; -i\langle\psi^\dagger\psi\rangle_E \;=\; -4N_c\int\frac{d^4k}{(2\pi)^4}\frac{M(k)}{k^2+M(k)^2}.
\tag{23}
$$

The subscripts M and E stand for Minkowski and Euclidean space, respectively. The gluon condensate is expressed as follows [13]:

$$\left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle \;=\; 32 N_c \int \frac{d^4k}{(2\pi)^4} \frac{M^2(k)}{k^2 + M^2(k)}. \tag{24}$$

It is already known that the imaginary part of the effective chiral action is identical to the Wess-Zumino-Witten action [36, 37] with the correct coefficient, which arises from the derivative expansion of the imaginary part to order $\mathcal{O}(p^5)$ [20, 38, 39, 40]. An appreciable merit of using the momentum-dependent quark mass as a regulator was already pointed out by Ball and Ripka [41]. The momentum-dependent quark mass provides a consistent regularization of the effective action given in Eq.(12) in which its real and imaginary parts are treated on the same footing and thus pertinent observables such as anomalous decays $\pi^0 \to 2\gamma$ are safely recovered even if $M(k)$ acts as a regulator.

III. EFFECTIVE WEAK CHIRAL ACTION

In this section, we will show how the effective weak Hamiltonian is incorporated into the present framework. The effective weak chiral action $S^{\Delta S=1,2}_{\rm eff}[\pi^a]$ can be written as follows:

$$\exp\left(-S^{\Delta S=1,2}_{\rm eff}\right) \;=\; \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \exp\left[\int d^4x \left(\psi^\dagger D \psi - \mathcal{H}^{\Delta S=1,2}_{\rm eff}\right)\right]. \tag{25}$$

Here the effective weak quark Hamiltonian $\mathcal{H}^{\Delta S=1}_{\rm eff}$ consists of ten four-quark operators :

$$\mathcal{H}^{\Delta S=1}_{\rm eff} \;=\; -\frac{G_F}{\sqrt{2}} V_{ud} V^*_{us} \sum_i c_i(\mu) \mathcal{Q}_i(\mu) + \text{h.c.}. \tag{26}$$

The G_F is the well-known Fermi constant and V_{ij} denote the Cabibbo-Kobayashi-Maskawa(CKM) matrix elements. The τ is their ratio given by $\tau = -V_{td}V^*_{ts}/V_{ud}V^*_{us}$. The Wilson coefficients $c_i(\mu)$ are defined as $c_i(\mu) = z_i(\mu) + \tau y_i(\mu)$. The functions $z_i(\mu)$ and $y_i(\mu)$ are the scale-dependent Wilson coefficients given at the scale of the μ. The $z_i(\mu)$ represent the CP-conserving part, while $y_i(\mu)$ stand for the CP-violating one. The four-quark operators $\mathcal{Q}_i$ contain the dynamic information of the weak transitions, being constructed by integrating out the vector bosons $W^\pm$ and Z and heavy quarks t, b, and c. The four-quark operators [9] are given by

$$\begin{aligned}
\mathcal{Q}_1 &= -4\left(s^\dagger_\alpha \gamma_\mu P_L u_\beta\right)\left(u^\dagger_\beta \gamma_\mu P_L d_\alpha\right), \quad \mathcal{Q}_2 = -4\left(s^\dagger_\alpha \gamma_\mu P_L u_\alpha\right)\left(u^\dagger_\beta \gamma_\mu P_L d_\beta\right), \\
\mathcal{Q}_3 &= -4\left(s^\dagger_\alpha \gamma_\mu P_L d_\alpha\right)\sum_{q=u,d,s}\left(q^\dagger_\beta \gamma_\mu P_L q_\beta\right), \quad \mathcal{Q}_4 = -4\left(s^\dagger_\alpha \gamma_\mu P_L d_\beta\right)\sum_{q=u,d,s}\left(q^\dagger_\beta \gamma_\mu P_L q_\alpha\right), \\
\mathcal{Q}_5 &= -4\left(s^\dagger_\alpha \gamma_\mu P_L d_\alpha\right)\sum_{q=u,d,s}\left(q^\dagger_\beta \gamma_\mu P_R q_\beta\right), \quad \mathcal{Q}_6 = -4\left(s^\dagger_\alpha \gamma_\mu P_L d_\beta\right)\sum_{q=u,d,s}\left(q^\dagger_\beta \gamma_\mu P_R q_\alpha\right), \\
\mathcal{Q}_7 &= -6\left(s^\dagger_\alpha \gamma_\mu P_L d_\alpha\right)\sum_{q=u,d,s}\left(q^\dagger_\beta \hat{Q} \gamma_\mu P_R q_\beta\right), \quad \mathcal{Q}_8 = -6\left(s^\dagger_\alpha \gamma_\mu P_L d_\beta\right)\sum_{q=u,d,s}\left(q^\dagger_\beta \hat{Q} \gamma_\mu P_R q_\alpha\right), \\
\mathcal{Q}_9 &= -6\left(s^\dagger_\alpha \gamma_\mu P_L d_\alpha\right)\sum_{q=u,d,s}\left(q^\dagger_\beta \hat{Q} \gamma_\mu P_L q_\beta\right), \quad \mathcal{Q}_{10} = -6\left(s^\dagger_\alpha \gamma_\mu P_L d_\beta\right)\sum_{q=u,d,s}\left(q^\dagger_\beta \hat{Q} \gamma_\mu P_L q_\alpha\right),
\end{aligned} \tag{27}$$

where $P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$ are the chiral projection operators and $\hat{Q} = \frac{1}{3}{\rm diag}(2,-1,-1)$ denote the quark charge matrix. The $\mathcal{Q}_1$ and $\mathcal{Q}_2$ come from the current-current diagrams, while

$\mathcal{Q}_3$ to $\mathcal{Q}_6$ [5, 6, 7] and $\mathcal{Q}_7$ to $\mathcal{Q}_{10}$ [8] are induced by QCD penguin and electroweak penguin diagrams, respectively. Note that only seven operators in Eq.(27) are independent. For example, we can express $\mathcal{Q}_4$, $\mathcal{Q}_9$, and $\mathcal{Q}_{10}$ as follows:

$$\mathcal{Q}_4 = -\mathcal{Q}_1 + \mathcal{Q}_2 + \mathcal{Q}_3, \quad \mathcal{Q}_9 = \frac{1}{2}\left(3\mathcal{Q}_1 - \mathcal{Q}_3\right), \quad \mathcal{Q}_{10} = \mathcal{Q}_2 + \frac{1}{2}\left(\mathcal{Q}_1 - \mathcal{Q}_3\right). \tag{28}$$

Under the chiral transformation $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ the four-quark operators $Q_2 - Q_1$, $\mathcal{Q}_{3,4,5,6}$ transform like $(8_L, 1_R)$. The $\mathcal{Q}_1 + \mathcal{Q}_2$, $\mathcal{Q}_{9,10}$ transform like the combination of $(8_L, 1_R)$ and $(27_L, 1_R)$, while the $\mathcal{Q}_{7,8}$ transform like $(8_L, 8_R)$.

The $\Delta S = 2$ effective weak Hamiltonian is expressed as [42, 43, 44, 45]

$$\mathcal{H}_{\mathrm{eff}}^{\Delta S=2} = -\frac{G_F^2 M_W^2}{16\pi^2}\mathcal{F}\left(\lambda_c, \lambda_t, m_c^2, m_t^2, M_W^2\right) b(\mu)\mathcal{Q}_{\Delta S=2}(\mu) + \mathrm{h.c.} \tag{29}$$

with

$$\mathcal{F} = \lambda_c^2\eta_1 S\left(\frac{m_c^2}{M_W^2}\right) + \lambda_t^2\eta_2 S\left(\frac{m_t^2}{M_W^2}\right) + 2\lambda_c\lambda_t\eta_3 S\left(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right) \tag{30}$$

and the parameters $\lambda_q = V_{qd}V_{qs}^*$ denote the pertinent relations of the CKM matrix elements with $q = u, c, t$. The functions S_i are the Inami-Lim functions [42, 46, 47], being obtained by integrating over electroweak loops and describing the $|\Delta S| = 2$ transition amplitude in the absence of strong interactions. The $b(\mu)$ is again the corresponding scale-dependent Wilson coefficient. The coefficients η_i represent the short-distance QCD corrections split off from the $b(\mu)$ [45]. The four-quark operator $\mathcal{Q}_{\Delta S=2}$ is written as

$$\mathcal{Q}_{\Delta S=2} = -4\left(s_\alpha^\dagger\gamma_\mu P_L d_\alpha\right)\left(s_\beta^\dagger\gamma_\mu P_L d_\beta\right). \tag{31}$$

Since the Fermi constant G_F is very small, one can expand Eq.(25) in powers of the G_F and keep the lowest order only. Then we can obtain the effective weak chiral Lagrangian

$$\mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2} = -\frac{1}{\mathcal{Z}}\int \mathcal{D}\psi\mathcal{D}\psi^\dagger\mathcal{H}_{\mathrm{eff}}^{\Delta S=1,2}\exp\left[\int d^4x\psi^\dagger D\psi\right]. \tag{32}$$

If you write a generic operator for the four-quark operator $\mathcal{Q}_i$ for a given i in Euclidean space such as

$$\mathcal{Q}_i(x) = -\psi^\dagger(x)\gamma_\mu P_{R,L}\Lambda_1\psi(x)\psi^\dagger(x)\gamma_\mu P_{R,L}\Lambda_2\psi(x), \tag{33}$$

where $\Lambda_{1,2}$ denote the flavor spin operators, then we can calculate the vacuum expectation value (VEV) of $\mathcal{Q}_i(x)$ as follows:

$$\begin{aligned}\langle\mathcal{Q}_i\rangle &= \frac{1}{\mathcal{Z}}\int \mathcal{D}\psi\mathcal{D}\psi^\dagger\,\mathcal{Q}_i(x)\exp\left[\int d^4z\psi^\dagger D\psi\right] \\ &= \frac{1}{\mathcal{Z}}\int d^4y\int\frac{d^4k}{(2\pi)^4}e^{ik(x-y)}\frac{\delta}{\delta J_\mu^{(1)}(x)}\frac{\delta}{\delta J_\mu^{(2)}(y)} \\ &\quad\times\exp\left[\int d^4z\left\langle z\left|\mathrm{tr}\ln\tilde{D}(J_1(z), J_2(z))\right|z\right\rangle\right]_{J_1=J_2=0} \\ &= L_i^{(1)} + L_i^{(2)}.\end{aligned} \tag{34}$$

Here, $\tilde{D}$ is

$$\tilde{D}(J_1(z), J_2(z)) = D + J_\alpha^{(1)}(z)\gamma_\alpha P_{R,L}\Lambda_1 + J_\beta^{(2)}(z)\gamma_\beta P_{R,L}\Lambda_2. \tag{35}$$

The $L_i^{(1)}$ and $L_i^{(2)}$ are given by

$$\begin{aligned}
L_i^{(1)} &= -N_c^2 \mathrm{tr}\left[\left\langle x\left|\frac{1}{D}\gamma_\mu P_{R,L}\Lambda_1\right|x\right\rangle\left\langle x\left|\frac{1}{D}\gamma_\mu P_{R,L}\Lambda_2\right|x\right\rangle\right]_i + \mathcal{O}\left(N_c\right) \\
&= -N_c^2 \mathrm{tr}\left[(A_1)_\mu (A_2)_\mu\right]_i + \mathcal{O}\left(N_c\right) \qquad\qquad i = 1,4,6,8,10\,, \qquad (36)\\
L_i^{(2)} &= N_c^2 \mathrm{tr}\left[\left\langle x\left|\frac{1}{D}\gamma_\mu P_{R,L}\Lambda_1\right|x\right\rangle\right]\mathrm{tr}\left[\left\langle x\left|\frac{1}{D}\gamma_\mu P_{R,L}\Lambda_2\right|x\right\rangle\right]_i + \mathcal{O}\left(N_c\right) \\
&= N_c^2 \mathrm{tr}\left[(A_1)_\mu\right]\mathrm{tr}\left[(A_2)_\mu\right]_i + \mathcal{O}\left(N_c\right) \qquad i = 2,3,5,7,9\,, \qquad (37)
\end{aligned}$$

where $\Lambda_{1,2}$ are the corresponding flavor matrices. Applying the derivative expansion to the operators $(A_{1,2})_\mu$ up to order $\mathcal{O}(\partial^2)$ they can be written as

$$\begin{aligned}
(A_{1,2})_\mu &= \sum_{n=0}^{\infty}\int\frac{d^4k}{(2\pi)^4}\left(\frac{1}{k^2+M^2(k)}\right)^{n+1} \\
&\times \Big[2i\left(1+2M\tilde{M}'\right)k_\alpha\partial_\alpha + M(\partial\!\!\!/ U^{\gamma_5}) + 2iM\tilde{M}'k_\alpha U^{-\gamma_5}(\partial_\alpha U^{\gamma_5}) \\
&+ \left(1+2M\tilde{M}'\right)\partial^2 + M\tilde{M}'\left(U^{-\gamma_5}(\partial^2 U^{\gamma_5}) + 2U^{-\gamma_5}(\partial_\alpha U^{\gamma_5})\partial_\alpha\right) \\
&+ 4(M\tilde{M}'' + \tilde{M}'^2)k_\alpha k_\beta\left(\partial_\alpha\partial_\beta + U^{-\gamma_5}(\partial_\alpha U^{\gamma_5})\partial_\beta + \frac{1}{2}U^{-\gamma_5}(\partial_\alpha\partial_\beta U^{\gamma_5})\right) \\
&+ 2\tilde{M}'^2 k_\alpha k_\beta(\partial_\alpha U^{-\gamma_5})(\partial_\beta U^{\gamma_5}) - i\tilde{M}'k_\alpha\left((\partial_\alpha\partial\!\!\!/ U^{\gamma_5}) + 2(\partial\!\!\!/ U^{\gamma_5})\partial_\alpha\right)\Big]^n \\
&\times \Big[-k\!\!\!/ - iMU^{-\gamma_5} - \tilde{M}'k_\gamma(\partial_\gamma U^{-\gamma_5}) + \frac{i}{2}\tilde{M}'(\partial^2 U^{-\gamma_5}) \\
&+ i\left(\tilde{M}'' - \frac{1}{2}\frac{\tilde{M}'^2}{M}\right)k_\gamma k_\delta(\partial_\gamma\partial_\delta U^{-\gamma_5})\Big]\gamma_\mu P_{\mathrm{L,R}}\Lambda_{1,2}, \qquad (38)
\end{aligned}$$

where M, $\tilde{M}'$ and $\tilde{M}''$ are the k-dependent functions defined in Eq.(18). Before we proceed to determine the low energy constants for the effective weak chiral Lagrangian, we want to discuss the role of the large N_c limit and related problems of the current conservation.

IV. THE $1/N_c$ EXPANSION AND CURRENT CONSERVATION

The large N_c expansion enters into two different places: Wilson coefficients and four-quark operators. The N_c dependence of the 6×6 anomalous dimension matrix γ in the basis of $Q_{1,2,3,4,5,6}$ was shown by Ref. [6, 25]. The operators $Q_{7,8,9,10}$ are not mixed with $Q_{1,2,3,4,5,6}$ [9], since the LO and NLO anomalous dimensions γ_{ij}, $i=1,\cdots 6$, $j=7,\cdots 10$ are zero. Up to $\mathcal{O}(1/N_c)$, we write the matrix of the anomalous dimension [6, 25, 26] as

$$\gamma_{ij} = \frac{1}{2}\frac{\alpha_{\mathrm{s}}}{\pi}\Gamma_{ij} \qquad (39)$$

with

$$\Gamma_{ij} = \begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{11}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 3 & \frac{N_f}{3} & 0 & \frac{N_f}{3} \\ 0 & 0 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & \frac{N_f}{3} & 0 & \left(\frac{N_f}{3} - 3N_c\right) \end{pmatrix}, \qquad (40)$$

where $\alpha_s(\sim \frac{1}{N_c})$ denotes the strong running coupling constant. In the strict large-N_c limit, all elements vanish except the anomalous dimension of the Q_6 operator γ_{66}. It means that the operator Q_2 is not mixed with the other operators $Q_i (i \neq 2)$ and Q_6 is only renormalized. Because of $\alpha_s \sim \frac{1}{N_c}$, The Wilson coefficients $c_i (i \neq 2)$ go to zero and c_2 becomes unity. Thus, in the strict large-N_c limit the operator Q_2 does not require any renormalization, so that it restores the original current-current form of quark charged weak interaction at $\mu = M_W$, *i.e.* the factorized form.

Strictly speaking, the Q_2 is the only contribution to the effective weak chiral Lagrangian in the large N_c limit and other operators must be treated as the NLO corrections according to the Wilson coefficients [26]. However, we will take the weak Hamiltonian as our starting point more practically without considering the large N_c behavior of the anomalous dimensions [11].

Since the momentum-dependent quark mass presents the nonlocal interaction between the Goldstone-bosons and quarks, the vector and axial-vector currents are known to be not conserved. Making the effective action gauge-invariant with some approximations, one is able to derive conserved currents [30, 31, 32, 33]. The conserved vector and axial-vector currents in Euclidean space with the momentum-dependent quark mass are written as follows [32, 33]:

$$V_\mu^a = \bar{\psi}\gamma_\mu\lambda^a\psi + i\langle\bar{\psi}|x\rangle\langle x|\sqrt{M_\mu}\lambda^a U^{\gamma_5}\sqrt{M}|\psi\rangle + i\langle\bar{\psi}|\sqrt{M}U^{\gamma_5}\lambda^a\sqrt{M_\mu}|x\rangle\langle x|\psi\rangle, \tag{41}$$

$$A_\mu^a = \psi^\dagger\gamma_\mu\gamma_5\lambda^a\psi + i\langle\psi^\dagger|x\rangle\langle x|\sqrt{M_\mu}\gamma_5\lambda^a U^{\gamma_5}\sqrt{M}|\psi\rangle - i\langle\psi^\dagger|\sqrt{M}U^{\gamma_5}\gamma_5\lambda^a\sqrt{M_\mu}|x\rangle\langle x|\psi\rangle,$$

where $\sqrt{M_\mu}$ denotes $d\sqrt{M}/dp_\mu$. Last two terms in V_μ^a and A_μ^a are required so that the currents can be conserved.

The pion decay constant f_π is related to the following transition matrix elements:

$$\langle 0|A_\mu^a(x)|\pi^b(p)\rangle = if_\pi p^\mu e^{ip\cdot x}\delta^{ab}, \tag{42}$$

where A_μ^a is defined in Eq.(41). The f_π^2 obtained from Eq.(42) is exactly the same as Eq.(20) [33], which indicates that the PCAC is well satisfied with the additional nonlocal term in Eq.(41). With the nonlocal terms turned off, we would end up with the Pagels–Stokar condition for f_π^2 [48]:

$$f_\pi^2(\mathrm{PS}) = \int \frac{d^4k}{(2\pi)^4}\frac{M^2 - \frac{1}{4}MM'k}{(k^2+M^2)^2}. \tag{43}$$

Although the pion decay constant given in Eq.(20) is the correct one with the momentum-dependent quark mass, we want to use the Pagels-Stokar condition for the normalization of the effective chiral Lagrangian for convenience. The reason lies in the fact that by using the Pagels-Stokar condition we need not consider the additional nonlocal part of the currents in deriving the VEV of the quark operator, since we obtain the same results as we use the conserved currents given in Eq.(41) and hence we reproduce precisely the large N_c results for the B_K factor as well as the LECs.

V. EFFECTIVE WEAK CHIRAL LAGRANGIAN

A. Leading order in the $1/N_c$ expansion

In order of $\mathcal{O}(p^0)$, only the operator $\mathcal{Q}_8$ has the nonvanishing term:

$$\langle\mathcal{Q}_8 + \mathcal{Q}_8^\dagger\rangle_{\mathcal{O}(p^0)} = 48N_c^2\mathcal{M}^2\langle U\lambda_6U^\dagger\hat{Q}\rangle. \tag{44}$$

However, since we are mainly interested in the LECs $g_{\underline{8}}$ and $g_{\underline{27}}$ of the effective weak chiral Lagrangian in LO, we proceed to calculate the NLO terms, *i.e.* those in $\mathcal{O}(p^2)$ order:

$$
\begin{aligned}
\langle \mathcal{Q}_1 + \mathcal{Q}_1^\dagger \rangle^{\mathcal{O}(N_c^2)}_{\mathcal{O}(p^2)} &= 16 N_c^2 \mathcal{K}^2 \left(-\frac{2}{5} \langle \lambda_6 L_\mu L_\mu \rangle + \frac{1}{3} t^{ik}_{jl} \langle \lambda^j_i L_\mu \rangle \langle \lambda^l_k L_\mu \rangle \right), &(45)\\
\langle \mathcal{Q}_2 + \mathcal{Q}_2^\dagger \rangle^{\mathcal{O}(N_c^2)}_{\mathcal{O}(p^2)} &= 16 N_c \mathcal{K}^2 \left(\frac{3}{5} \langle \lambda_6 L_\mu L_\mu \rangle + \frac{1}{3} t^{ik}_{jl} \langle \lambda^j_i L_\mu \rangle \langle \lambda^l_k L_\mu \rangle \right), &(46)\\
\langle \mathcal{Q}_3 + \mathcal{Q}_3^\dagger \rangle^{\mathcal{O}(N_c^2)}_{\mathcal{O}(p^2)} &= 0, &(47)\\
\langle \mathcal{Q}_4 + \mathcal{Q}_4^\dagger \rangle^{\mathcal{O}(N_c^2)}_{\mathcal{O}(p^2)} &= 16 N_c^2 \mathcal{K}^2 \langle \lambda_6 L_\mu L_\mu \rangle, &(48)\\
\langle \mathcal{Q}_5 + \mathcal{Q}_5^\dagger \rangle^{\mathcal{O}(N_c^2)}_{\mathcal{O}(p^2)} &= 0, &(49)\\
\langle \mathcal{Q}_6 + \mathcal{Q}_6^\dagger \rangle^{\mathcal{O}(N_c^2)}_{\mathcal{O}(p^2)} &= 64 N_c^2 \mathcal{M} \left(\mathcal{P} + \mathcal{R} \right) \langle \lambda_6 L_\mu L_\mu \rangle, &(50)\\
\langle \mathcal{Q}_7 + \mathcal{Q}_7^\dagger \rangle^{\mathcal{O}(N_c^2)}_{\mathcal{O}(p^2)} &= 24 N_c^2 \mathcal{K}^2 \langle L_\mu \lambda_6 \rangle \langle R_\mu \hat{Q} \rangle, &(51)\\
\langle \mathcal{Q}_8 + \mathcal{Q}_8^\dagger \rangle^{\mathcal{O}(N_c^2)}_{\mathcal{O}(p^2)} &= 48 N_c^2 \mathcal{M} \left(\mathcal{P} + \mathcal{R} \right) \Big[\langle U \lambda_6 \left(\partial_\mu U^\dagger \right) \left(\partial_\mu U \right) U^\dagger \hat{Q} \rangle \\
&+ \langle \left(\partial_\mu U \right) \left(\partial_\mu U^\dagger \right) U \lambda_6 U^\dagger \hat{Q} \rangle \Big], &(52)\\
\langle \mathcal{Q}_9 + \mathcal{Q}_9^\dagger \rangle^{\mathcal{O}(N_c^2)}_{\mathcal{O}(p^2)} &= 16 N_c^2 \mathcal{K}^2 \left(-\frac{3}{5} \langle \lambda_6 L_\mu L_\mu \rangle + \frac{1}{2} t^{ik}_{jl} \langle \lambda^j_i L_\mu \rangle \langle \lambda^l_k L_\mu \rangle \right), &(53)\\
\langle \mathcal{Q}_{10} + \mathcal{Q}_{10}^\dagger \rangle^{\mathcal{O}(N_c^2)}_{\mathcal{O}(p^2)} &= 16 N_c^2 \mathcal{K}^2 \left(\frac{2}{5} \langle \lambda_6 L_\mu L_\mu \rangle + \frac{1}{2} t^{ik}_{jl} \langle \lambda^j_i L_\mu \rangle \langle \lambda^l_k L_\mu \rangle \right), &(54)\\
\langle \mathcal{Q}_{\Delta S=2} + \mathcal{Q}_{\Delta S=2}^\dagger \rangle^{\mathcal{O}(N_c^2)}_{\mathcal{O}(p^2)} &= 16 N_c^2 \mathcal{K}^2 \langle \lambda_6 L_\mu \lambda_6 L_\mu \rangle, &(55)
\end{aligned}
$$

where t^{ik}_{jl} are the eikosiheptaplet projection operators [10, 49] and the functions $\mathcal{K}$, $\mathcal{M}$, $\mathcal{P}$, and $\mathcal{R}$ are expressed as follows:

$$
\begin{aligned}
\mathcal{K} &= \int \frac{d^4k}{(2\pi)^4} \frac{M^2 - \frac{1}{4} M M' k}{\left(k^2 + M^2\right)^2} = f_\pi^2(\mathrm{PS}),\\
\mathcal{M} &= -\int \frac{d^4k}{(2\pi)^4} \frac{M}{k^2 + M^2} = \frac{\langle \bar{\psi}\psi \rangle_M}{4N_c},\\
\mathcal{P} &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{\frac{M'^2}{32M}}{k^2 + M^2} + \frac{M - \frac{1}{4} M M'^2}{\left(k^2 + M^2\right)^2} + \frac{-\frac{1}{2} M k^2 - \frac{1}{2} M^2 M' k + \frac{1}{4} M^3 M'^2}{\left(k^2 + M^2\right)^3} \right],\\
\mathcal{R} &= \int \frac{d^4k}{(2\pi)^4} \frac{\frac{1}{2} M^2 M' k - M^3}{\left(k^2 + M^2\right)^3}. &(56)
\end{aligned}
$$

Note that the function $\mathcal{K}$ is just the same as the Pagels-Stokar condition and the function $\mathcal{M}$ is directly related to the quark condensate. The $\Delta S = 1$ effective weak chiral Lagrangian [50] is given in Minkowski space as follows:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{eff}}^{\Delta S=1, \mathcal{O}(p^2)} &= -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* f_\pi^4 \left(g_{\underline{8}} \mathcal{L}_{\underline{8}} + g_{\underline{27}} \mathcal{L}_{\underline{27}} \right)\\
&= -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* f_\pi^4 \left(g_{\underline{8}}^{(1/2)} \mathcal{L}_{\underline{8}}^{(1/2)} + g_{\underline{27}}^{(1/2)} \mathcal{L}_{\underline{27}}^{(1/2)} + g_{\underline{27}}^{(3/2)} \mathcal{L}_{\underline{27}}^{(3/2)} \right), &(57)
\end{aligned}
$$

where

$$
\mathcal{L}_{\underline{8}} = \left\langle \lambda_3^2 L_\mu L^\mu \right\rangle + \text{h.c.},
$$

$$\begin{aligned}\mathcal{L}_{\underline{27}} &= \frac{2}{3}\left\langle \lambda_1^2 L_\mu \right\rangle \left\langle \lambda_1^3 L^\mu \right\rangle + \left\langle \lambda_2^3 L_\mu \right\rangle \left\langle \lambda_1^1 L^\mu \right\rangle \; + \; \mathrm{h.c.} \\ &= \frac{1}{9}\mathcal{L}_{\underline{27}}^{(1/2)} \; + \; \frac{5}{9}\mathcal{L}_{\underline{27}}^{(3/2)},\end{aligned} \tag{58}$$

and

$$\begin{aligned}\mathcal{L}_{\underline{8}}^{(1/2)} &= \left\langle \lambda_2^3 L_\mu L^\mu \right\rangle \; + \; \mathrm{h.c.}, \\ \mathcal{L}_{\underline{27}}^{(1/2)} &= \left\langle \lambda_1^2 L_\mu \right\rangle \left\langle \lambda_3^1 L^\mu \right\rangle - \left\langle \lambda_3^2 L_\mu \right\rangle \left\langle \lambda_1^1 L^\mu \right\rangle - 5\left\langle \lambda_3^2 L_\mu \right\rangle \left\langle \lambda_3^3 L^\mu \right\rangle + \mathrm{h.c.}, \\ \mathcal{L}_{\underline{27}}^{(3/2)} &= \left\langle \lambda_1^2 L_\mu \right\rangle \left\langle \lambda_3^1 L^\mu \right\rangle + 2\left\langle \lambda_3^2 L_\mu \right\rangle \left\langle \lambda_1^1 L^\mu \right\rangle + \left\langle \lambda_3^2 L_\mu \right\rangle \left\langle \lambda_3^3 L^\mu \right\rangle + \mathrm{h.c.}.\end{aligned} \tag{59}$$

The $g_{\underline{8}}$ and $g_{\underline{27}}$ are dimensionless LECs of which the numerical values can be extracted from the CP-conserving $K \to \pi\pi$ decay rate and the $\Delta T = 1/2$ enhancement is reflected in these constants. From the analysis in chiral perturbation theory with chiral loops considered [51, 52], the LECs have the following values

$$|g_{\underline{8}}|_{\mathrm{exp}} \simeq 3.6, \quad |g_{\underline{27}}|_{\mathrm{exp}} \simeq 0.29, \quad \frac{|g_{\underline{8}}|_{\mathrm{exp}}}{|g_{\underline{27}}|_{\mathrm{exp}}} \simeq 12.5, \tag{60}$$

Comparing Eq.(57) with Eqs.(45-54), we determine the values of the LECs $g_{\underline{8}}$ and $g_{\underline{27}}$ to the LO in the $1/N_c$ expansion:

$$\begin{aligned}g_{\underline{8}} &= -\frac{2}{5}c_1 + \frac{3}{5}c_2 + c_4 - \frac{3}{5}c_9 + \frac{2}{5}c_{10} + \frac{64 N_c^2 \mathcal{M}(\mathcal{P}+\mathcal{R})}{f_\pi^4} c_6, \\ g_{\underline{27}} &= \frac{3}{5}c_1 + \frac{3}{5}c_2 + \frac{9}{10}c_9 + \frac{9}{10}c_{10}.\end{aligned} \tag{61}$$

In the strict large N_c limit in which the Wilson coefficient c_2 survives only and becomes one, we correctly reproduce the following large N_c results [51]:

$$\begin{aligned}g_{\underline{8}}\Big|_{N_c\to\infty} &= \frac{3}{5}, \\ g_{\underline{27}}\Big|_{N_c\to\infty} &= \frac{3}{5}.\end{aligned} \tag{62}$$

The effective $\Delta S = 2$ weak chiral Lagrangian to order $\mathcal{O}(p^2)$ is derived as:

$$\mathcal{L}_{\mathrm{eff}}^{\Delta S=2,\mathcal{O}(p^4)} = -\frac{G_F^2 M_W^2}{4\pi^2}\mathcal{F}\left(\lambda_c, \lambda_t, m_c^2, m_t^2, M_W^2\right)\frac{4}{3}f_\pi^4 \hat{B}_K \langle \lambda_6 L_\mu\rangle\langle \lambda_6 L^\mu\rangle, \tag{63}$$

where $\hat{B}_K$ is known as the scale-independent B_K factor which is defined as $B_K = B_K(\mu)b(\mu)$. The scale-dependent $B_K(\mu)$ is related to the matrix element for the $\bar{K}^0 - K^0$ mixing

$$\langle \bar{K}^0 | \mathcal{Q}_{\Delta S=2} | K^0 \rangle = \frac{16}{3} B_K(\mu) f_K^2 m_K^2, \tag{64}$$

which governs the $\bar{K}^0 - K^0$ mixing at short distances. Here, f_K and m_K are the mass and decay constant of the neutral kaon, respectively. Comparing Eq.(63) with Eq.(55) in the chiral limit ($f_\pi = f_K$), we immediately obtain the B_K factor in the large N_c limit [53, 54]:

$$B_K = \frac{3}{4}. \tag{65}$$

B. $\mathcal{O}(N_c)$ corrections

Now, we proceed to add the next-to-order corrections in the large N_c expansion. Though there are many different origins of the NLO corrections in the N_c expansion, we are not able to consider all possible NLO corrections. Here, we will restrict ourselves the $1/N_c$ corrections from the quark-quark operators. The VEVs of the quark operators in the NLO are as follows:

$$\begin{aligned}
\langle \mathcal{Q}_1 + \mathcal{Q}_1^\dagger \rangle^{\mathcal{O}(N_c)}_{\mathcal{O}(p^2)} &= 16 N_c \mathcal{K}^2 \left(\frac{3}{5} \langle \lambda_6 L_\mu L_\mu \rangle + \frac{1}{3} t^{ik}_{jl} \langle \lambda^j_i L_\mu \rangle \langle \lambda^l_k L_\mu \rangle \right), && (66)\\
\langle \mathcal{Q}_2 + \mathcal{Q}_2^\dagger \rangle^{\mathcal{O}(N_c)}_{\mathcal{O}(p^2)} &= 16 N_c \mathcal{K}^2 \left(-\frac{2}{5} \langle \lambda_6 L_\mu L_\mu \rangle + \frac{1}{3} t^{ik}_{jl} \langle \lambda^j_i L_\mu \rangle \langle \lambda^l_k L_\mu \rangle \right), && (67)\\
\langle \mathcal{Q}_3 + \mathcal{Q}_3^\dagger \rangle^{\mathcal{O}(N_c)}_{\mathcal{O}(p^2)} &= 16 N_c \mathcal{K}^2 \langle \lambda_6 L_\mu L_\mu \rangle, && (68)\\
\langle \mathcal{Q}_4 + \mathcal{Q}_4^\dagger \rangle^{\mathcal{O}(N_c)}_{\mathcal{O}(p^2)} &= 0, && (69)\\
\langle \mathcal{Q}_5 + \mathcal{Q}_5^\dagger \rangle^{\mathcal{O}(N_c)}_{\mathcal{O}(p^2)} &= 64 N_c \mathcal{M} \left(\mathcal{P} + \mathcal{R} \right) \langle \lambda_6 L_\mu L_\mu \rangle, && (70)\\
\langle \mathcal{Q}_6 + \mathcal{Q}_6^\dagger \rangle^{\mathcal{O}(N_c)}_{\mathcal{O}(p^2)} &= 0, && (71)\\
\langle \mathcal{Q}_7 + \mathcal{Q}_7^\dagger \rangle^{\mathcal{O}(N_c)}_{\mathcal{O}(p^2)} &= 48 N_c \mathcal{M} \left(\mathcal{P} + \mathcal{R} \right) \Big[\langle U \lambda_6 \left(\partial_\mu U^\dagger \right) \left(\partial_\mu U \right) U^\dagger \hat{Q} \rangle \\
&+ \langle \left(\partial_\mu U \right) \left(\partial_\mu U^\dagger \right) U \lambda_6 U^\dagger \hat{Q} \rangle \Big], && (72)\\
\langle \mathcal{Q}_8 + \mathcal{Q}_8^\dagger \rangle^{\mathcal{O}(N_c)}_{\mathcal{O}(p^2)} &= 24 N_c \mathcal{K}^2 \langle L_\mu \lambda_6 \rangle \langle R_\mu \hat{Q} \rangle, && (73)\\
\langle \mathcal{Q}_9 + \mathcal{Q}_9^\dagger \rangle^{\mathcal{O}(N_c)}_{\mathcal{O}(p^2)} &= 16 N_c \mathcal{K}^2 \left(\frac{2}{5} \langle \lambda_6 L_\mu L_\mu \rangle + \frac{1}{2} t^{ik}_{jl} \langle \lambda^j_i L_\mu \rangle \langle \lambda^l_k L_\mu \rangle \right), && (74)\\
\langle \mathcal{Q}_{10} + \mathcal{Q}_{10}^\dagger \rangle^{\mathcal{O}(N_c)}_{\mathcal{O}(p^2)} &= 16 N_c \mathcal{K}^2 \left(-\frac{3}{5} \langle \lambda_6 L_\mu L_\mu \rangle + \frac{1}{2} t^{ik}_{jl} \langle \lambda^j_i L_\mu \rangle \langle \lambda^l_k L_\mu \rangle \right), && (75)\\
\langle \mathcal{Q}_{\Delta S=2} + \mathcal{Q}_{\Delta S=2}^\dagger \rangle^{\mathcal{O}(N_c)}_{\mathcal{O}(p^2)} &= 16 N_c \mathcal{K}^2 \langle \lambda_6 L_\mu \lambda_6 L_\mu \rangle. && (76)
\end{aligned}$$

The LECs $g_{\underline{8}}$ and $g_{\underline{27}}$ are then obtained as follows:

$$\begin{aligned}
g_{\underline{8}} &= \left(-\frac{2}{5} + \frac{3}{5N_c} \right) c_1 + \left(\frac{3}{5} - \frac{2}{5N_c} \right) c_2 + \frac{1}{N_c} c_3 + c_4 + \left(-\frac{3}{5} + \frac{2}{5N_c} \right) c_9 + \left(\frac{2}{5} - \frac{3}{5N_c} \right) c_{10} \\
&+ \frac{64 N_c \mathcal{M}(\mathcal{P}+\mathcal{R})}{f_\pi^4} c_5 + \frac{64 N_c^2 \mathcal{M}(\mathcal{P}+\mathcal{R})}{f_\pi^4} c_6, \\
g_{\underline{27}} &= \left(1 + \frac{1}{N_c} \right) \left(\frac{3}{5} c_1 + \frac{3}{5} c_2 + \frac{9}{10} c_9 + \frac{9}{10} c_{10} \right). && (77)
\end{aligned}$$

If we turn off the momentum dependence of the $M(k)$ and introduce a regularization with the cut-off parameter Λ for the quark-loop integrals, we end up with the former results in Ref. [10]:

$$\begin{aligned}
\mathcal{K} &= \frac{f_\pi^2}{4N_c}, \\
\mathcal{M} &= \frac{\langle \bar{\psi}\psi \rangle}{4N_c}, \\
\mathcal{P} &= \frac{f_\pi^2}{8N_c M} + \frac{M}{64\pi^2},
\end{aligned}$$

$$\mathcal{R} \;=\; -\frac{M}{32\pi^2}. \tag{78}$$

Thus, the LECs (77) are reduced to those with the constant constituent quark mass [10]:

$$\begin{aligned}
g_{\underline{8}} \;=\; & \left(-\frac{2}{5}+\frac{3}{5N_c}\right)c_1+\left(\frac{3}{5}-\frac{2}{5N_c}\right)c_2+\frac{1}{N_c}c_3+c_4+\left(-\frac{3}{5}+\frac{2}{5N_c}\right)c_9+\left(\frac{2}{5}-\frac{3}{5N_c}\right)c_{10} \\
& +\left(\frac{2\langle\bar{\psi}\psi\rangle}{N_c f_\pi^2 M}-\frac{\langle\bar{\psi}\psi\rangle M}{4f_\pi^4\pi^2}\right)c_5+\left(\frac{2\langle\bar{\psi}\psi\rangle}{f_\pi^2 M}-\frac{N_c\langle\bar{\psi}\psi\rangle M}{4f_\pi^4\pi^2}\right)c_6, \\
g_{\underline{27}} \;=\; & \left(1+\frac{1}{N_c}\right)\left(\frac{3}{5}c_1+\frac{3}{5}c_2+\frac{9}{10}c_9+\frac{9}{10}c_{10}\right).
\end{aligned} \tag{79}$$

From Eq.(76), the B_K factor with the $\mathcal{O}(N_c)$ correction becomes

$$B_K \;=\; \frac{3}{4}\left(1+\frac{1}{N_c}\right), \tag{80}$$

which is just one with $N_c = 3$ [55] and agrees with the result of $1/N_c$ approach [54, 56].

VI. RESULTS AND DISCUSSION

We employed the Wilson coefficients c_i obtained by Buchalla *et al.* [9]. There are three different renormalization schemes in Ref. [9]. For our best fit, we choose the naive dimensional regularization (NDR) scheme in this work.

While in the former calculation based on the usual chiral quark model [10] three parameters are involved, namely the pion decay constant, the quark condensate, and the constituent quark mass, the present results given by Eqs.(61,77) include four functions $\mathcal{K}$, $\mathcal{M}$, $\mathcal{P}$, and $\mathcal{R}$. However, those functions depend on the $M(k)$ containing the Λ and M_0, so that we have only two free parameters. Furthermore, the cut-off parameter Λ is fixed by reproducing the pion decay constant in our calculation. The value of M_0 is more or less constrained by demanding the calculated quark condensate (23) and gluon condensate (24) to lie inside the empirical limits of $-(350\text{ MeV})^3 \leq \langle\bar{\psi}\psi\rangle \leq -(200\text{ MeV})^3$ [57, 58, 59] and $(350\text{ MeV})^4 \leq \langle\frac{\alpha_s}{\pi}GG\rangle \leq (400\text{ MeV})^4$ [60, 61, 62]. Thus, we first want to examine the dependence of the quark and gluon condensates on the momentum-dependent quark mass (see Eqs.(23,24)).

In Figure 2 we show the dependence of the quark condensate on the M_0 with several different types of the $M(k)$ given in Eq.(10). The $M(k)$ by Diakonov and Petrov [12, 13] and that of the dipole type produce the very similar results of the $\langle\bar{\psi}\psi\rangle$ [63], the Gaussian type brings out noticeably smaller value of the quark condensate than the other two. From these results one can easily see that the $M(k)$ with stronger tail produces the larger value of the quark condensate. The shaded band depicts the empirical range of the quark condensate.

In Figure 3 we draw the dependence of the gluon condensate on the M_0 in the same manner. As in the case of the quark condensate, the dependence on the different types of the $M(k)$ is similar. Again, the shaded band represents the empirical range of the gluon condensate. Thus, examining the dependence of the quark and gluon condensates, one constrains the range of the value of M_0.

In Figures 4 and 5 we present dependence of the LECs $g_{\underline{8}}$ and $g_{\underline{27}}$ on the dynamical quark mass, with the NDR scheme employed ($\Lambda^{(4)}_{\bar{M}S} = 435$ MeV). As seen in Eqs.(61,77), the $g_{\underline{27}}$

does not depend on M_0. On the other hand, the $g_{\underline{8}}$ shows a strong dependence on the M_0. It is due to the fact that the penguin operator contributes only to the $g_{\underline{8}}$ and it brings about three functions containing the M_0 and the quark condensate. The behavior of the $g_{\underline{8}}$ with different types of $M(k)$ is similar to the quark and gluon condensates. In particular, the $g_{\underline{8}}$ goes up drastically below $M_0 = 250$ MeV. As a result, the ratio $g_{\underline{8}}/g_{\underline{27}}$ exhibits a similar dependence on the M_0 as shown in Fig. 6.

Figures 7 and 8 depict the contribution of the NLO corrections $\mathcal{O}(N_c)$ to the LECs $g_{\underline{8}}$ and $g_{\underline{27}}$. The $\mathcal{O}(N_c)$ contribution to the $g_{\underline{8}}$ is very tiny, while it does to the $g_{\underline{27}}$ almost 30% and thus it makes the $g_{\underline{27}}$ deviate from the empirical value. As already discussed in Ref.[10], the $\mathcal{O}(N_c)$ corrections make the ratio $g_{\underline{8}}/g_{\underline{27}}$ worse as shown in Fig. 6, compared to the empirical value (dotted line). We only can reach the empirical value below around $M_0 = 230$ MeV which is not preferable according to the proper range of the M_0 from the quark and gluon condensates.

Now, we want to analyze the contributions to the LECs from the different quark operators Q_i given in Eq.(27). These considerations are basically independent of the actual form of $M(k)$. Since the penguin operator Q_6 contributes only to the octet part of the coupling, it contributes directly to the $\Delta T = 1/2$ enhancement. In fact, the appearance of this octet penguin operator indicates already that the effect of perturbative gluons is of utmost importance in explaining the $\Delta T = 1/2$ rule. Figure 9 shows each contribution of the quark operators Q_i, starting from the Q_2 which has the same form of the bare operator at the scale of the M_W. The operator Q_2 has the largest Wilson coefficient in any renormalization scheme. The Q_1 takes the second largest Wilson coefficient. As drawn in Fig. 9, the LEC $g_{\underline{8}}$ is much underestimated with the operators Q_1 and Q_2 only and furthermore they are stable to the change of the M_0. Adding the penguin operator Q_6 brings the $g_{\underline{8}}$ up dramatically and its dependence on the M_0 is also very noticeable. The other contributions are negligibly tiny. Hence, the penguin operator Q_6 plays an essential role in enhancing the octet coupling. Actually, considering the fact that the $M(k)$ is pertinent to the zero mode of individual instantons, the strong dependence of the penguin operator on the M indicates the importance of the effect of a part of nonperturbative gluons in describing the nonleptonic kaon decays.

In the case of the $g_{\underline{27}}$, the operators Q_1 and Q_2 are dominant and the Wilson coefficients c_9 and c_{10} are negligibly small.

VII. SUMMARY AND CONCLUSIONS

In the present work we have investigated the effective weak chiral Lagrangian for $\Delta S = 1, 2$ concentrating on the calculation of $g_{\underline{8}}$ and $g_{\underline{27}}$. We used the chiral quark model from the instanton vacuum as framework and incorporated the weak interaction by the effective Hamiltonian of Buchalla, Buras, and Lautenbacher [9]. The calculation has been done in a first step to order $\mathcal{O}(p^2)$ and to LO and NLO in the $1/N_c$ expansion. In contrast to the previous work by Ref. [10], we used a momentum-dependent constituent quark mass $M(k)$ as it arises from the instanton vacuum, which makes $g_{\underline{8}}$ be closer to its corresponding empirical value. However, the $g_{\underline{27}}$ is untouched by the present work, so that it is not at all improved, since it is independent of M.

For their ratio $g_{\underline{8}}/g_{\underline{27}}$, this simple feature enlarged the values by about 50 % to 100 % and hence very much improved the theoretical values compared with the empirical ones. In fact we tried various shapes of $M(k)$. However, the best values were obtained for the $M(k)$ from

the instanton model of Diakonov and Petrov [12, 13] and $M_0 \simeq 230$ MeV. Altogether the empirical value of $\underline{g_8}/\underline{g_{27}}$ with the M_0 constrained to the quark and gluon condensates is still underestimated and one is still away from an explanation of the $\Delta T = 1/2$ rule. However, the conclusion is very clear: If one wishes to derive the effective weak Lagrangian from the instanton vacuum of QCD by means of the chiral quark model the use of a momentum-dependent quark mass seems to be indispensable.

Acknowledgments

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Figures

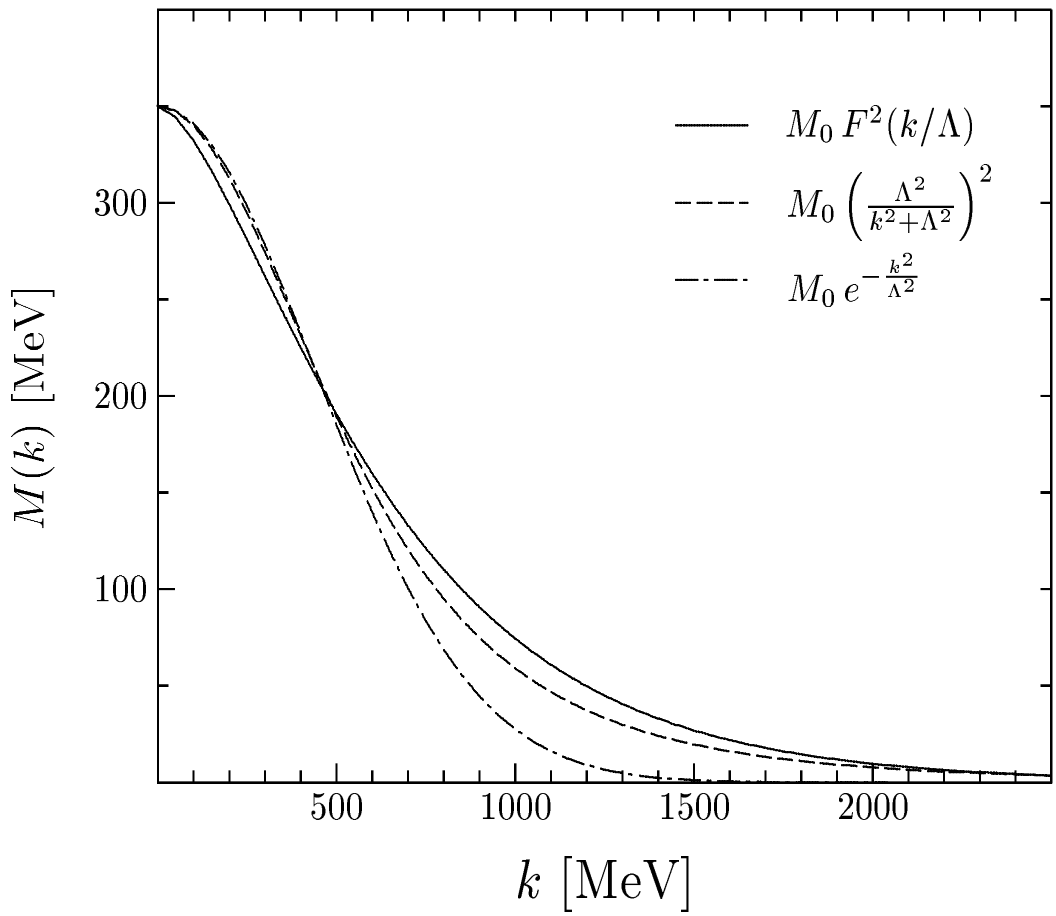

Fig.1: Momentum dependence of the constituent quark mass $M(k)$. The solid curve denotes the original $M(k) = M_0F^2(k/\Lambda)$ by Diakonov and Petrov, while the long-dashed one stands for the dipole-type $M(k)$ and the dot-dashed one depicts the Gaussian one. The Λ has been fitted to reproduce $f_\pi = 93$ MeV.

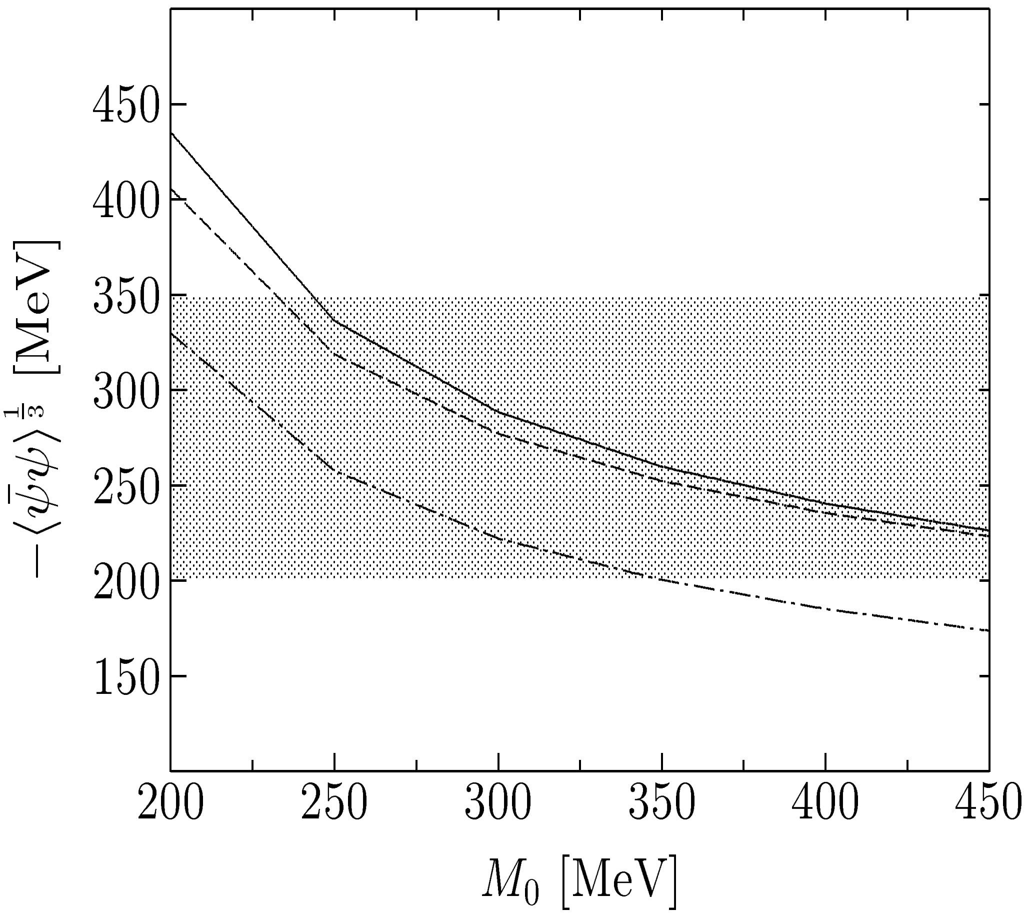

Fig.2: Quark condensate as a function of $M(k)$. The solid curve denotes the original $M(k) = M_0 F^2(k/\Lambda)$ by Diakonov and Petrov, while the long-dashed one stands for the dipole-type $M(k)$ and the dot-dashed one depicts the Gaussian one. The shaded band designates the range of the empirical values of the quark condensate.

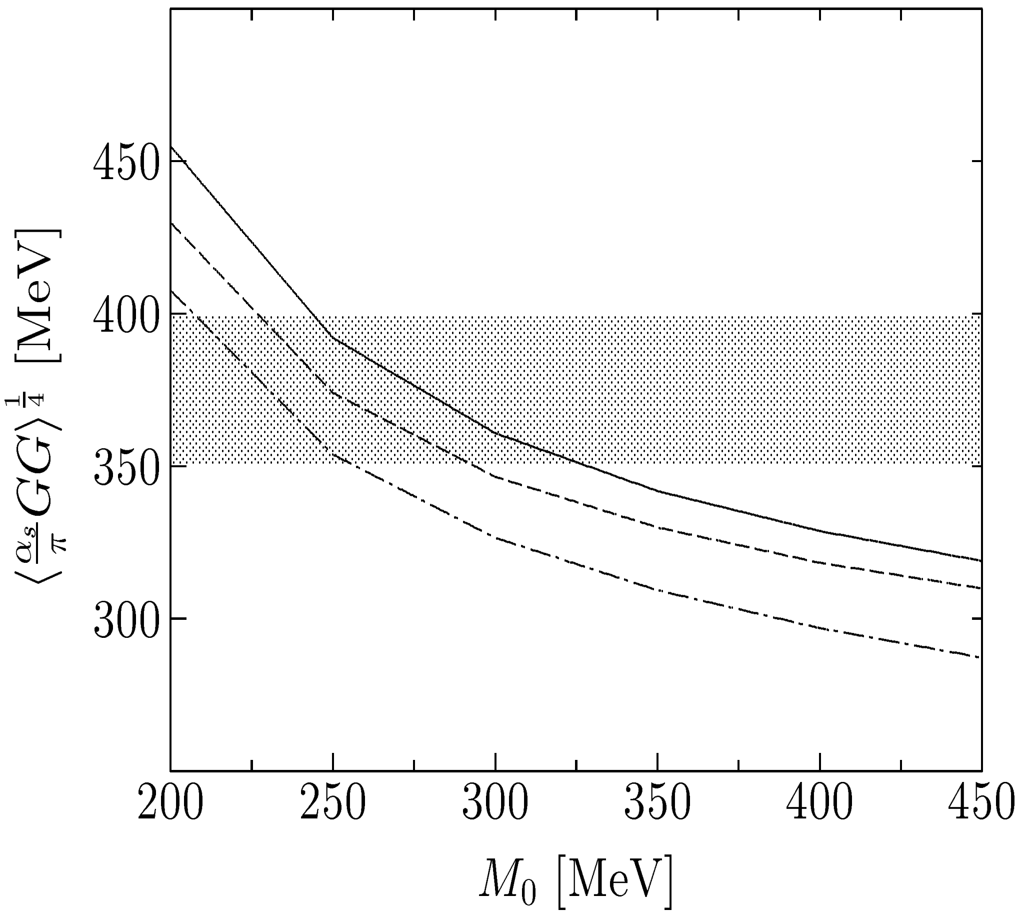

Fig.3: Gluon condensate as a function of $M(k)$. The solid curve denotes the original $M(k) = M_0 F^2(k/\Lambda)$ by Diakonov and Petrov, while the long-dashed one stands for the dipole-type $M(k)$ and the dot-dashed one depicts the Gaussian one. The shaded band designates the range of the empirical values of the gluon condensate.

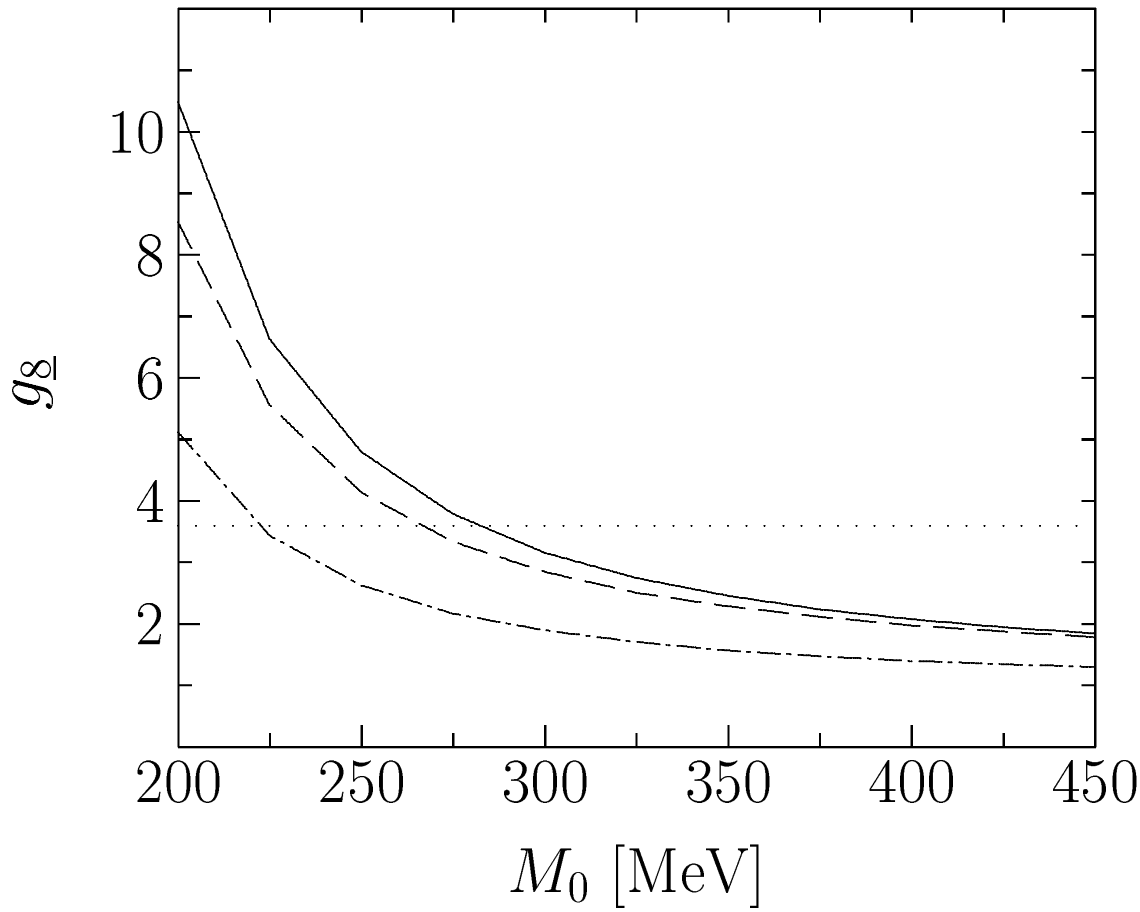

Fig.4: The low energy constant $g_{\underline{8}}$ as a function of $M(k)$. The solid curve denotes the original $M(k) = M_0F^2(k/\Lambda)$ by Diakonov and Petrov, while the long-dashed one stands for the dipole-type $M(k)$ and the dot-dashed one is the Gaussian one. The empirical value of $|g_{\underline{8}}|$ is approximately 3.6 which is drawn in the dotted line.

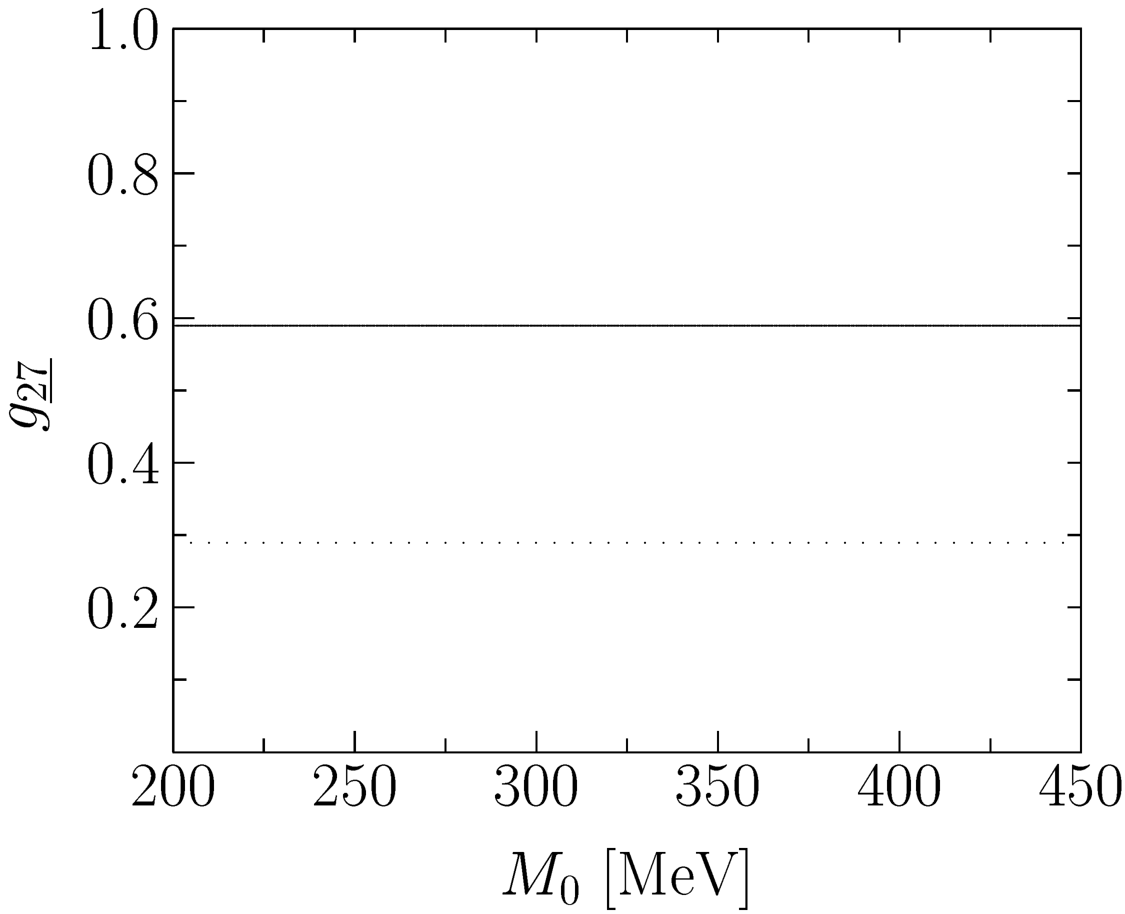

Fig.5: The low energy constant $g_{\underline{27}}$ as a function of $M(k)$. It is independent of the $M(k)$.

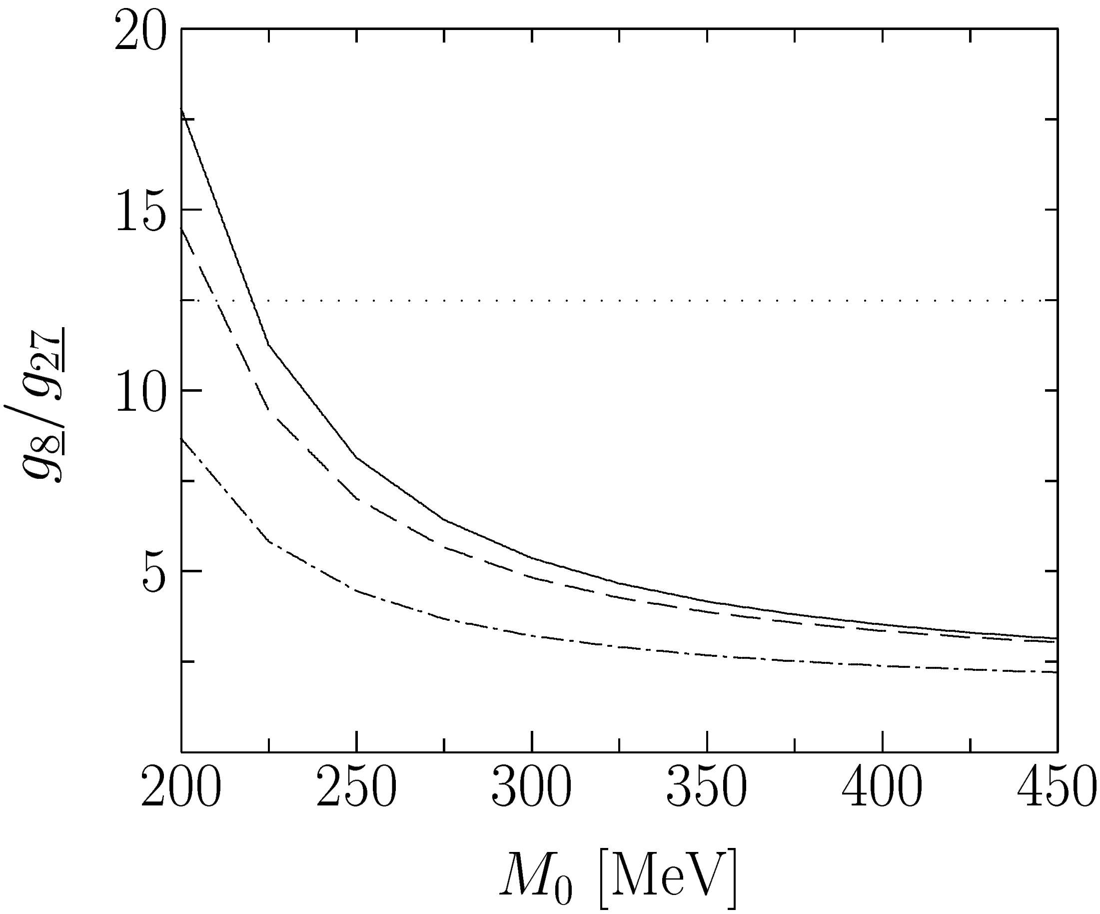

Fig.6: The ratio $g_{\underline{8}}/g_{\underline{27}}$ as a function of $M(k)$. The solid curve denotes the original $M(k) = M_0 F^2(k/\Lambda)$ by Diakonov and Petrov, while the long-dashed one stands for the dipole-type $M(k)$ and the dot-dashed one is the Gaussian one. The empirical value of $|g_{\underline{8}}/g_{\underline{27}}|$ is approximately 12.5 which is drawn in the dotted line.

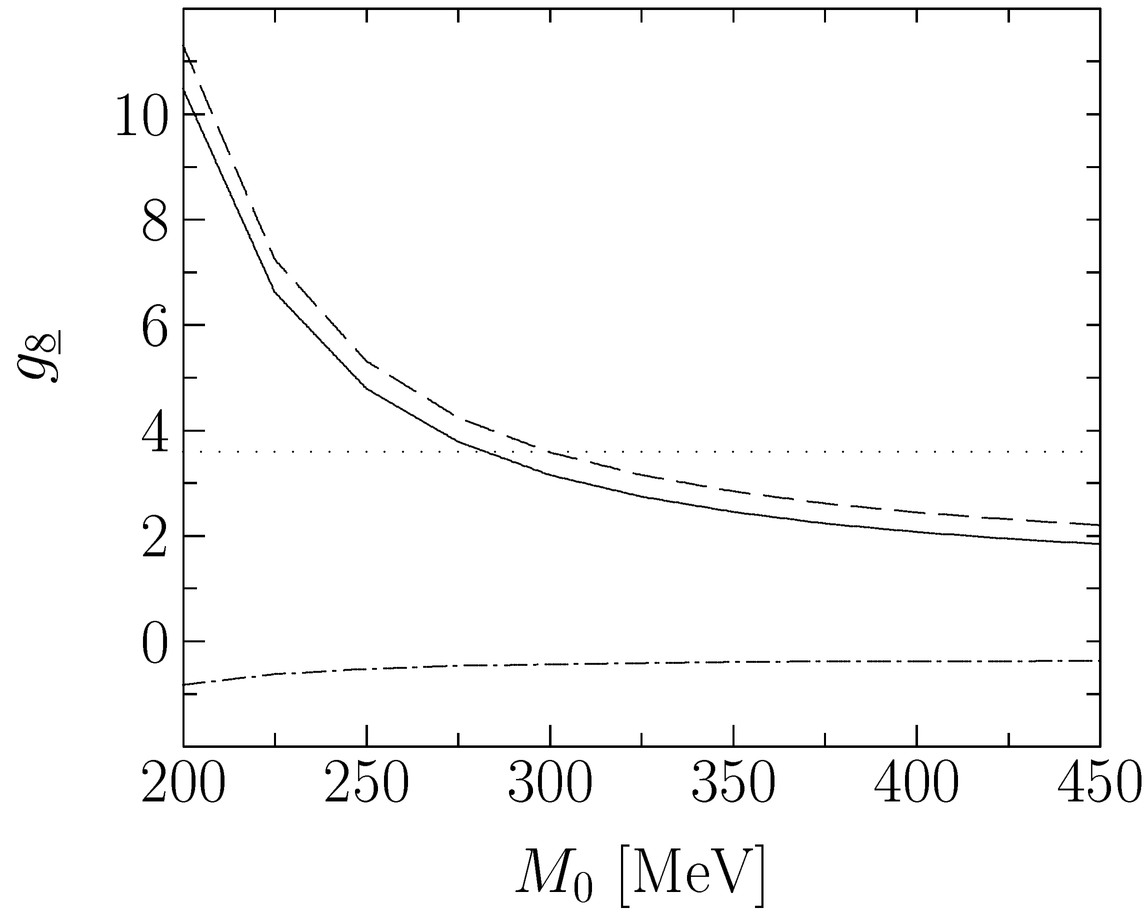

Fig. 7 The LO and NLO contributions to the $g_{\underline{8}}$ in the large N_c expansion. The long-dashed curve represents the LO contribution, the dot-dashed one the NLO contribution, and the solid one the full result. The empirical value of $|g_{\underline{8}}|$ is approximately 3.6 which is drawn in the dotted line.

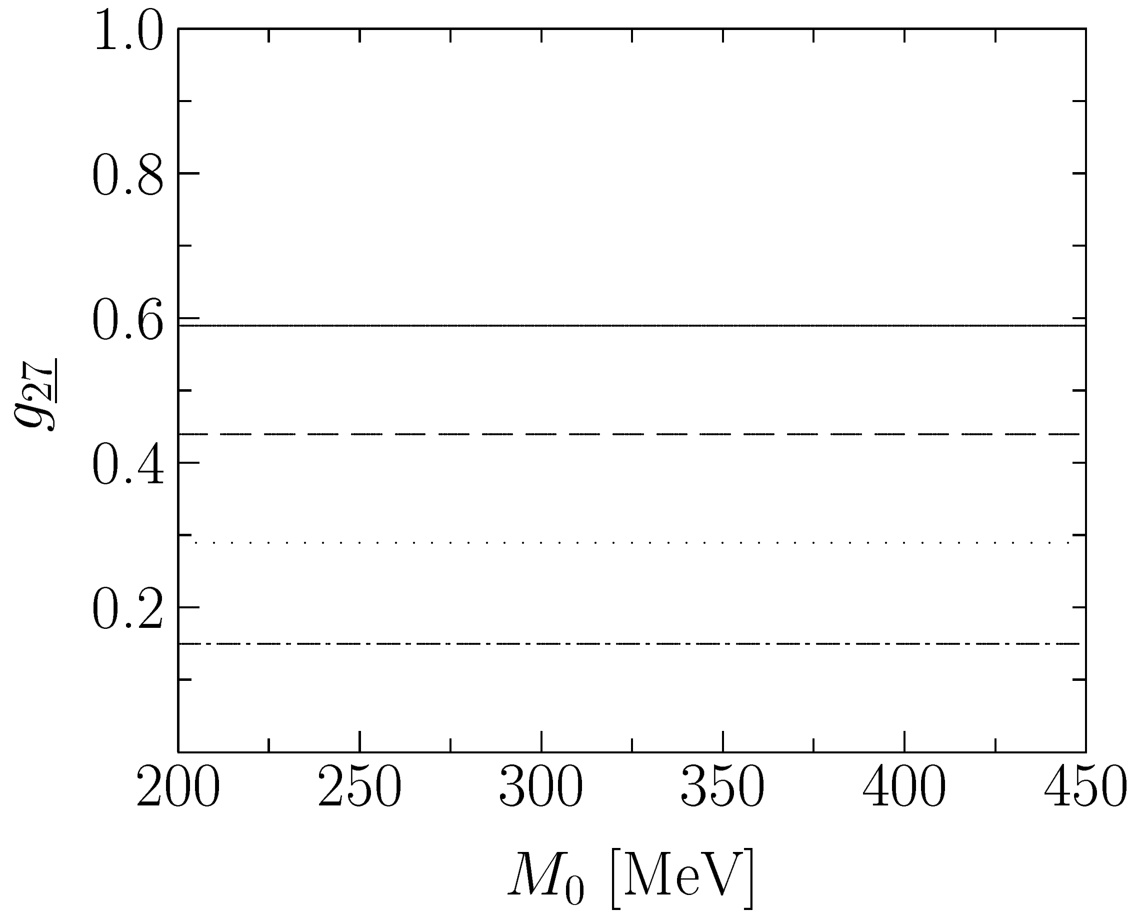

Fig. 8 The LO and NLO contributions to the $g_{\underline{27}}$ in the large N_c expansion. The long-dashed curve represents the LO contribution, the dot-dashed one the NLO contribution, and the solid one the full result. The empirical value of $|g_{\underline{27}}|$ is approximately 0.29 which is drawn in the dotted line.

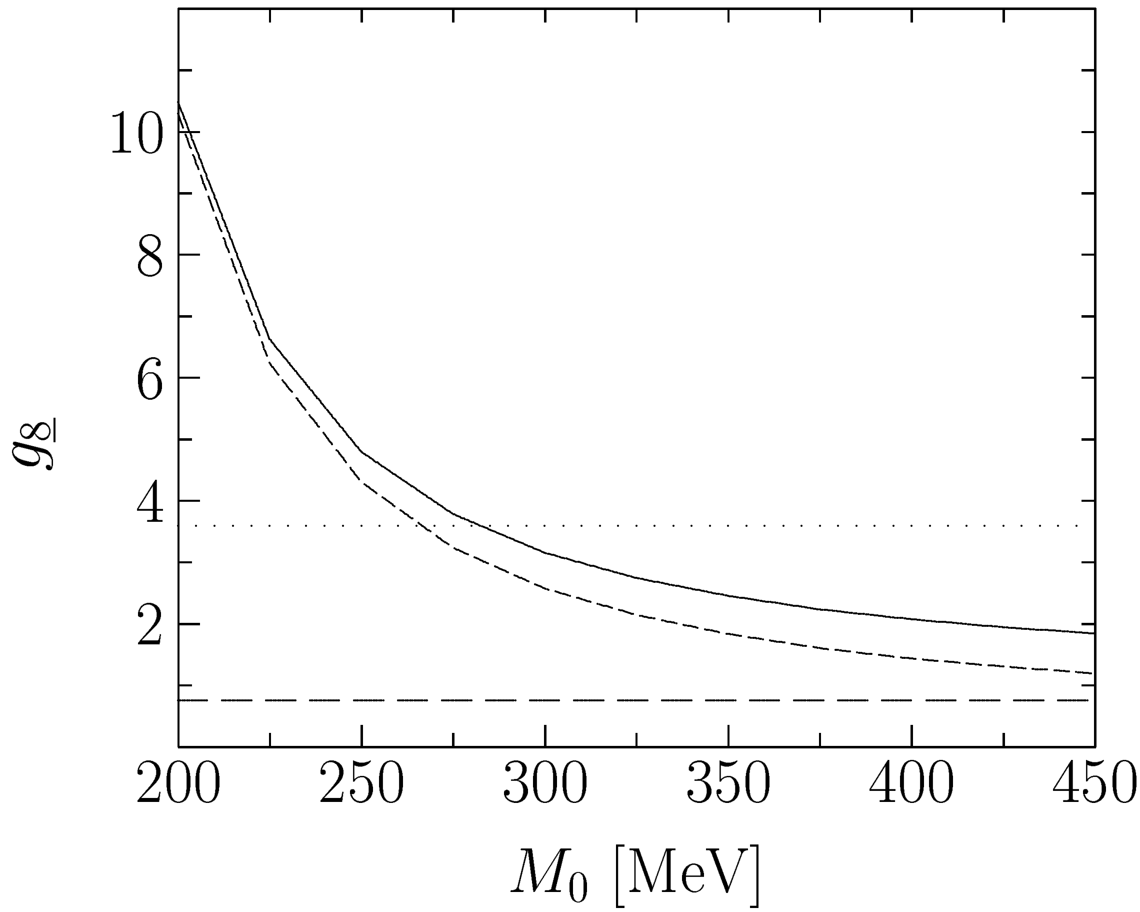

Fig.9: Contributions of each quark operator Q_i to the low energy constant $g_{\underline{8}}$ as a function of $M(k)$. The long-dashed curve denotes the contribution of the Q_2, while the short-dashed one draws that of the $Q_1 + Q_2$. The dot-dashed one depicts the $g_{\underline{8}}$ with Q_6 added and the solid curve represents the full result. The empirical value of $|g_{\underline{8}}|$ is approximately 3.6 which is drawn in the dotted line.